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INFINITE SET OF CONSERVATION LAWS OF THE QUANTUM CHIRAL FIELD IN TWO-DIMENSIONAL SPACE-TIME

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БЕСКОНЕЧНОЕ ЧИСЛО КВАНТОВЫХ ЗАКОНОВ СОХРАНЕНИЯ ДВУМЕРНОГО КИРАЛЬНОГО ПОЛЯ

Аннотация

Рассматривается квантованное $O(N)$ -ковариантное киральное поле в двумерном пространстве-времени в рамках $1/N$ теории возмущений, свободной от ультрафиолетовых и инфракрасных расходимостей. Построено бесконечное число сохраняющихся токов полиномиальных по полю на основе квантовых уравнений движения и уравнений группы перенормировок. Это дает динамическое обоснование гипотезы о факторизации оператора рассеяния в модели.

Abstract

The quantized $O(N)$ -covariant chiral field in two-dimensional space-time is considered in the $1/N$ perturbation theory free of ultraviolet and infrared divergences. An infinite set of conserved local currents polynomial in the chiral field is constructed applying quantum equations of motion and renormalization group equations. This provides a dynamical justification of the recent conjecture about the factorization of the S -matrix in the theory.

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0. Introduction. We consider N -component chiral field in two space-time dimensions: $n(x)=\{n_1(x),\dots,n_N(x)\}$, $x=(t,x)$. Chiral fields play important role in the construction of nontrivial models in the theory of elementary particles and statistical physics [1,2]. The classical Lagrangian and the subsidiary condition are given by:

$$\mathcal{L}(x)=1/2\,(\partial_\xi n,\partial_\eta n) \tag{0.1}$$

$$n^2(x)=N/\gamma \tag{0.2}$$

where ξ and η are the light-cone variables $\xi=(t-x)/2$, $\eta=(t+x)/2$, $\partial_\xi=\partial_t-\partial_x$, $\partial_\eta=\partial_t+\partial_x$, $(\ ,\ )$ - denotes the scalar product in $\mathbb{R}^N$, γ is dimensionless coupling constant. The classical equations of motion written down with the aid of the Lagrange multiplier $\sigma(x)$ have the form:

$$\partial^2_{\xi\eta}n+\sigma n=0,\quad \sigma=-\gamma/N\,(n,\partial^2_{\xi\eta}n) \tag{0.3}$$

The Lagrangian (0.1) and the subsidiary condition (0.2) define a completely integrable system. Conformal invariance implies the existence of an infinite set of conserved local, though nonpolynomial with respect to the n -field currents [3]. In particular, the following equation holds:

$$\partial_\xi(\partial_\eta n,\partial_\eta n)=\partial_\eta(\partial_\xi n,\partial_\xi n)=0 \tag{0.4}$$

The theory of the quantized chiral field (0.1)-(0.2) free from ultraviolet and infrared divergences in each order of the $1/N$ perturbation theory expansion was developed in [4-6]. The leading order in the $1/N$ expansion was a subject of a whole series of works [7].

Interesting features of the $1/N$ perturbation theory are: 1. dimensional transmutation resulting from the renormalization: each component $n_i(x)$ becomes massive. 2. absence of multiple production and factorization of the scattering amplitudes in the leading order of the $1/N$ expansion [8].

The main purpose of this note is to construct an infinite set of conserved local currents in the quantum chiral theory which are polynomial in the n -field. This implies the conservation of the sets of initial and final momenta and the absence of multiple production in the scattering process in all orders of the renormalized $1/N$ perturbation theory. The existence of these currents also provides a dynamical justification of the recent conjecture about the factorization of the S -matrix *). The same situation was obseved in the sine-Gordon model [10,11].

The quantum currents are defined as composite operators in the Zimmermann's normal product formalism [12]. The basic tool to analize them are the quantum equations of motion for composite operators [13] which substantially differ from the classical equations of motion,since "anomalous" terms appear in the former having the structure of composite operators with anisotropic oversubtractions. We note,that the infinite set of conservation laws in the quantized sine-Gordon [14] and massive Thirring [15,16] models was proved to exist applying the same method. In these papers all "anomalies" resulting from the renormalization rearranged into quantum corrections to the coefficients of the classical expressions for the conserved currents **). However, in the chiral theory, there are no classical polynomial counterparts to the quantum conserved currents. Dimensional transmutation (and, therefore, breaking of conformal invariance) implies that the only polynomial conservation law (0.4) does not hold. Higher (nonpolynomial) classical currents of [3] are also meaningless,since no term in them can be defined as a composite operator.

Here, following [14-16] we shall look for conserved currents which obey the equations:

$$\partial_\eta \langle \mathcal{N}[J_\xi^{(2k+1)}](x)X\rangle + \partial_\xi \langle \mathcal{N}[J_\eta^{(2k+1)}](x)X\rangle =$$
$$-i\sum_{\ell=1}^{L}\sum_{A=1}^{M(k)} h_A\,\delta(x-x_\ell)\langle \mathcal{N}[Q^A](x)\hat{X}^\ell\rangle$$
$$X=\prod_{\ell=1}^{L} n_{i_\ell}(x_\ell) \qquad \hat{X}^\ell=\prod_{\ell'=1,\ell'\neq\ell}^{L} n_{i_{\ell'}}(x_{\ell'}) \tag{0.5}$$
$$J_\xi^{(2k+1)} = J_0^{(2k+1)} - J_1^{(2k+1)}, \quad J_\eta^{(2k+1)} = J_0^{(2k+1)} + J_1^{(2k+1)}, \quad k=0,1,2,\ldots$$

*) The conclusion that the infinite set of conservation laws implies factorization is supported by the investigation of some nonrelativistic models [9].

**) Later on appeared the papers [17] in which the results of [14-16] were rederived.

Here $\mathcal{N}[B](x)$ denotes normal product of the field monomial $B(x)$ defined for a minimum number of subtractions [12], $\{h_A\}$ is a finite set of numbers and $\langle\ldots\rangle$ are connected time-ordered Green's functions. The r.h.s. of eg.0.5) expresses the so called "contact terms" (or, covariant Schwinger terms) [13]*).

The currents $J_\xi^{(2k+1)}$ have the structure of $\xi\ldots\xi$ -component of a Lorentz tensor of rank $r=2k+2$ and canonical dimension $\dim J_\xi^{(2k+1)} = 2k+2$. $J_\eta^{(2k+1)}$ is the $\xi\ldots\xi$ -component of a Lorentz tensor of rank $r'=2k+1$ and $\dim J_\eta^{(2k+1)}=2k+2$.

As shown in [14-16] the integrated over x contact terms in eq.(0.5), after applying to them the Lehmann-Symanzik-Zimmermann (LSZ) reduction formulae [18], result in the conservation law for the $2k+1$-th momentum power of the initial and final scattering states **)

$$\sum_{\ell=1}^{L}(p_{\ell,\mu})^{2k+1}\langle out|in\rangle = 0 \tag{0.6}$$

We construct the conserved quantum chiral currents after the following scheme: $J_\xi^{(2k+1)}(x)$ is considered as a linear combination of composite operators all of which have Lorentz structure and dimension as stated above:

$$\mathcal{N}[J_\xi^{(2k+1)}](x) = \sum_{b=1}^{m(k)} c_b\,[\partial_\xi n]^b_{(2k+2)}(x) \tag{0.7}$$

where $[\partial_\xi n]^b_{(2k+2)}(x)$ denotes a composite operator which is only n -field monomial with $2k+2$ derivatives with respect to ξ. Using the quantum eqs. of motion [13] and Zimmermann's identities [12] we express the derivative with respect to η of each term in the form

$$\partial_\eta[\partial_\xi n]^b_{(2k+2)}(x) = \sum_a d_{ba}\,\partial_\xi[\partial_\xi n;\sigma]^a_{(2k+2)}(x) + \sum_{d=1}^{g(k)} f_{bd}\,[\partial_\xi n;\sigma]^d_{(2k+1)}(x) + \text{contact terms} \tag{0.8}$$

*) There exists a new series of conserved currents corresponding to the symmetry $\xi \rightleftarrows \eta$.

**) (0.6) equivalently reads: $[S,\hat{Q}^{(2k+1)}]=0$, $\hat{Q}^{(2k+1)} = \int dx\, J_0^{(2k+1)}(x)$

When $k=0,1,2$ $\rho(k)<m(k)$, therefore choosing appropriate C_β one can always arrive at $\sum_\beta c_\beta f_{\beta\alpha}=0$ for all $\alpha=1,\dots,\rho(k)$. Thus the existence of (0.5) is proved in this case.

However, $\rho(k)$ grows faster than $m(k)$ with the increase of k : $\rho(k)>m(k)$, $k\geqslant 3$, consequently, additional information about the second term in the r.h.s. of (0.8) in required to prove the existence of higher conservation laws. It is obtained by the analysis of the renormalization group equations for composite operators which imply that $\sum_{\beta=1}^{m(k)} c_\beta \sum_{\alpha=1}^{\rho(k)} f_{\beta\alpha} \langle [\partial_\xi n;\sigma]^\alpha_{(2k+1)}(x) X\rangle$ can be expressed in terms of total derivatives and contact terms.

The paper is organized as follows: in section 1. the necessary graph language in the $1/N$ perturbation theory for chiral field is reviewed, renormalization procedure with partially "soft" mass and the chiral condition are formulated. In section 2. the quantum equation of motion and the renormalization group equations for the Green functions with arbitrary composite operator insertions are derived. In section 3. the conserved chiral currents for $k=0,1,2$ are considered. Section 4. is devoted to the construction of higher conserved currents on the basis of renormalization group equations.

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1. $1/N$ Renormalized Perturbation Theory.

We shall remind here the necessary information for the $1/N$ renormalized perturbation theory of the n-field [5,4]. The generating functional for the Green's functions is:

$$Z[J]=\int\prod_x d\sigma \exp\left[-N/2\, S_1[\sigma]+i/2\int d^2x\, d^2y\,(J(x),(\Box+\sigma)^{-1}J(y))\right]$$

$$S_1[\sigma]=\mathrm{Tr}\,\ln(\Box+\sigma)+i/\gamma\int d^2x\,\sigma \tag{1.1}$$

where the integration over the n-field has been performed. The $1/N$ perturbation theory is constructed by expansion of the

effective σ-field action around the stationary point $\sigma_c=m^2$ of $S_1[\sigma]$. With the shift $\sigma=\sigma_c+\frac{\tilde\sigma}{\sqrt N}$ (1.1) reads explicitely:

$$Z[J]=\int\prod_x d\tilde\sigma \exp\Big\{N/2\sum_{k\geqslant 1}\mathcal{F}^{(k)}[\sigma]+1/2\sum_{k\geqslant 0}\int dx\,dy\,(J(x),G^{(k)}[\sigma](x,y)J(y))\Big\} \tag{1.2}$$

$$\mathcal{F}^{(1)}[\tilde\sigma]=\int dx\,\tilde\sigma/\sqrt N\,(i/\gamma-\mathcal{D}_c(0,m^2))=0 \tag{1.2a}$$

$$\mathcal{F}^{(k)}[\tilde\sigma]=(-\sqrt N)/k\int dx_1\dots dx_k\,\mathcal{D}_c(x-x_1,m^2)\dots\mathcal{D}_c(x_k-x_1,m^2)\tilde\sigma(x_1)\dots\tilde\sigma(x_k) \tag{1.2b}$$

$$\mathcal{D}_c(x,m^2)=\int d^2k\,e^{-ikx}/(2\pi)^2 i\,(m^2-k^2-i0)\,, \tag{1.2c}$$

(1.2a) is the (unrenormalized) stationarity equation for $S_1[\sigma]$. The Bogoliubov-Parasiuk-Hepp-Zimmermann's (BPHZ) subtraction scheme [20] with partially "soft" mass [21] is applied to define the renormalized one-particle irreducible both in n and $\tilde\sigma$ lines graphs (1PI) of (1.2). All graphs with (nonlocal) vertices $\mathcal{F}^{(k)}[\tilde\sigma]$, $k\geqslant 2$ and $G^{(k)}[\tilde\sigma]$, $k\geqslant 1$ are renormalized with "hard" mass m, while the divergent pointloop $\mathcal{D}_c(0,m^2)$ in $\mathcal{F}^{(1)}[\tilde\sigma]$ is subtracted with "soft" mass m around the subtraction mass μ. Note, that the latter leads to the renormalized stationary equation for $S_1[\sigma]$ (1.2a): $m^2=\mu^2 e^{-4\pi/\gamma}$ which is exactly the dimensional transmutation.

The graph elements are represented in fig 1. Here $\sum(p^2)$ is defined by:

$$\Sigma(p^2)=-\int d^2k/(2\pi)^2(m^2-k^2-i0)(m^2-(p-k)^2-i0)$$

In this tecnique the following types of graphs are not allowed for: tadpoles of one n-line pointloop and insertions into the $\tilde\sigma$-line of one loop graphs built out of two n-lines. Vertices $iNa(\gamma)\tilde\sigma/2$ and $ib(\gamma)(\partial_\mu n)^2/2$ account for the finite counterterms added to fit the normalization conditions:

$$\Gamma^{(2)}_{(p^2)}\Big|_{p^2=\mu^2} = i(m^2-\mu^2), \quad \partial/\partial p^2\, \Gamma^{(2)}_{(p^2)}\Big|_{p^2=\mu^2} = -i \qquad (1.3)$$

Here $\Gamma^{(2)}$ is the two-point 1PI with respect only to the n -lines Green's function.

The Green's functions constructed in each order of the expansion after this graphical scheme obey the conditions of Lorentz covariance, locality and unitarity as well as the quantum chiral condition (1.5).[5]. Unitarity follows from the Cutkosky rules [22] with cuts of n - and $\tilde{\sigma}$-lines as shown in fig.2 [4]. The quantum chiral condition (1.5) is an analogue of the classical subsidiary condition (0.2) and is formulated is the Zimmermann's normal product formalism.

We shall use subtraction procedure with "hard" mass in the construction of composite operators. Let the composite operator P is a monomial in n, $\tilde{\sigma}$ and their derivatives. The canonical dimension d of P equals the sum of the number of derivatives and two times the number of $\tilde{\sigma}$ -fields in P. Then the normal product $\mathcal{N}_d[P]$ is defined by:

$$\langle \mathcal{N}_d[P](x)X\rangle = \mathrm{f.p.}\Big\{\int \prod_x dn\, d\sigma\, P(x) X \times$$

$$\exp i/2 \int d^2x \big[(1+b(\gamma))(\partial_\mu n)^2 - (\tilde{\sigma}/\sqrt{N} + m^2)(n^2 - N/\gamma - Na(\gamma))\big]\Big\} \qquad (1.4)$$

where the finite part f.p. is evaluated with the help of the well known "forest" formula [20].

With the difinition (1.4) the quantum chiral condition looks like:

$$\mathcal{N}_d[Pn^2](x) = Na(\gamma)\mathcal{N}_d[P](x) \qquad (1.5)$$

In the following we shall also use the Zimmermann's identities [12]. They connect equal type of monomials P which have different Subtraction schemes in the subgraphs, containing lines of P. For example:

$$\mathcal{N}_\delta[P] - \mathcal{N}_d[P] = \sum_{\delta > d_i > d} r_i \mathcal{N}_{d_i}[P_i], \quad d_i = \dim P_i$$

The coefficients r_i are given explicitely in terms of 1PI Green's functions.

2. <u>Quantum Equations of Motion and Renormalization Group Equations of the Chiral Field</u>. In this section we apply the method proposed in [13] to derive the quantum equations of motion for the n-field. The analysis of the graphs shown in fig.3 gives:

$$(1+b)\langle \mathcal{N}[P\partial_x^\rho(n_{\xi\eta})](x)X\rangle = -\langle \mathcal{N}[P\partial_x^\rho(n\sigma)](x)X\rangle +$$

$$+m^2\langle \mathcal{N}[P\partial_x^\rho(n-\{n\})](x)X\rangle - i\sum_{l=1}^{L}\partial_x^\rho\,\delta(x-x_l)\langle \mathcal{N}[P](x)\hat{X}^l\rangle \qquad (2.1)$$

where the operator product X was introduced in (0.5), ∂_x^ρ is differential operator in x of order ρ, $\mathcal{N}[P\partial_x^\rho\{n\}]$ denotes anisotropic normal product *) in which two oversubtractions correspond to the n -line. Relations (2.1) are called quantum equations of motion.

Renormalization group equations are obtained following the method of [19]. For this purpose we introduce three operations of composite vertex insertions in the graphs for the connected Green's functions:

$$\Delta_0 = -i/\sqrt{N}\int dx\,\tilde{\sigma}(x), \qquad \Delta_1 = i/2\int dx\,\mathcal{N}[(n_{\xi\eta}, n)](x) \qquad (2.2)$$

$$\Delta_2 = -i/\sqrt{N}\int dx\,\{\tilde{\sigma}(x)\}$$

Here $\{\ \}$ denotes two oversubtractions.

Differentiating with respect to m^2 and μ^2 and applyind the well known "forest" formula [20] we obtain the following two relations for the renormalized connected Green's functions $\langle \mathcal{N}[P_\alpha]X\rangle$ with L external n-lines:

$$\partial/\partial m^2\langle \mathcal{N}[P_\alpha]X\rangle = (N/8\pi m^2\Delta_2 - N/2\,\partial a/\partial m^2 - \partial b/\partial m^2\,\Delta_1)\langle \mathcal{N}[P_\alpha]X\rangle \qquad (2.3)$$

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*) Anisotropic normal products were introduced in the most general form in the second ref. [13].

$$\partial/\partial\mu^2 \langle \mathcal{N}[P_\alpha](x)X\rangle = -\left(N/2\,\frac{\partial a}{\partial\mu^2}\Delta_0 + \frac{\partial b}{\partial\mu^2}\Delta_1\right)\langle \mathcal{N}[P_\alpha](x)X\rangle \tag{2.4}$$

Here the set of P_α includes all composite operators of equal Lorentz, $O(N)$ structure and canonical dimension. Two more relations between the operations of composite vertex insertions Δ_0, Δ_1, Δ_2 are implied by: a) the quantum equations of motion (2.1):

$$(1+b)\Delta_1\langle \mathcal{N}[P_\alpha]X\rangle = (N\Delta_2/8\pi + Na\Delta_0/2 + (L+|P_\alpha|)/2)\langle \mathcal{N}[P_\alpha]X\rangle \tag{2.5}$$

$|P_\alpha|$ is the number of n-lines in P_α ; b) the Zimmermann's identities (1.6) which applied in our case give (fig.4.):

$$(\Delta_2-\Delta_0)\langle \mathcal{N}[P_\alpha]X\rangle = -(\alpha_0\Delta_0+\alpha_1\Delta_1)\langle \mathcal{N}[P_\alpha]X\rangle -$$

$$-\sum_\beta \zeta'_{\alpha\beta}\langle \mathcal{N}[P_\beta]X\rangle + \text{contact terms} \tag{2.6}$$

The coefficients $\zeta'_{\alpha\beta}$ depend explicitely on the set of P_α while α_0 and α_1 do not and are given by:

$$\alpha_0 = \frac{iN}{8\pi m^2}\sum_{r\geq 1}(a/2)^r\Delta_0\Delta_0\langle(\tilde{n}(0),\tilde{n}(0))^r\rangle^{prop} - 8i\pi m^2/N\,\tilde{\Gamma}'_{\sigma\sigma}(0)$$

$$\alpha_1 = \sum_{r\geq 0}(a/2)^r\, i/N\,\partial/\partial p^2\,\Delta_0\langle(\tilde{n}(p),\tilde{n}(-p))(\tilde{n}(0),\tilde{n}(0))^r\rangle^{prop}\Big|_{p=0} \tag{2.7}$$

$$\tilde{\Gamma}'_{\sigma\sigma}(0) = -1/N\,\langle\widetilde{\sigma\sigma}\rangle^{prop}\Big|_{p=0}$$

The superscript "prop" used in (2.7) denotes the amputated one-particle irreducible (in n and $\tilde{\sigma}$-lines) Green's function, "$\sim$" is the Fourier transform. The matrix $\zeta'_{\alpha\beta}$ up to multiplicative factor represents the matrix of anomalous dimensions $\zeta_{\alpha\beta}$ corresponding to the set P_α. Contact terms in (2.6) appear only in case of σ operators in the monomials P_α.

Now we express the operations Δ_0, Δ_1 and Δ_2 from eqs.(2.3)-(2.5) and substitute the result into (2.6) to obtain the renormalization group equations:

$$[\mu^2\partial/\partial\mu^2 + (1+4\pi/\gamma^2\,\beta(\gamma))m^2\partial/\partial m^2 + \zeta(\gamma)(L+|P_\alpha|)]\langle \mathcal{N}[P_\alpha]X\rangle = \tag{2.8}$$

$$= -\sum \zeta_{\alpha\beta}(\gamma)\langle \mathcal{N}[P_\beta]X\rangle + \text{contact terms}$$

where $\beta(\gamma)$ and $\zeta(\gamma)$ are the usual β-function and the anomalous dimension of the n-field, respectively. We remind that, due to the dimensional transmutation, γ is considered as the function $\gamma^{-1} = (4\pi)^{-1}\ln\mu^2/m^2$.

3. Chiral conserved currents $J^{(2k+1)}$ for $k=0,1,2$.

A. First of all we consider the consevation of quantum energy momentum tensor [*]. Using quantum equations of motion (2.1), the chirality condition (1.5) and the obvious relation $\mathcal{N}[\{n^2\}] = \mathcal{N}[(n,\{n\})]$ we obtain:

$$1/2\,\partial_\eta\langle \mathcal{N}[n_\xi^2](x)X\rangle = m^2/1+b\,\langle \mathcal{N}[(n_\xi, n-\{n\})(x)X\rangle + \text{contact terms} =$$

$$= -m^2/2(1+b)\,\partial_\xi\langle \mathcal{N}[\{n^2\}](x)X\rangle + \text{contact terms} \tag{3.1}$$

Zimmermann's identities and equations of motion imply (fig.5):

$$\langle \mathcal{N}[\{n^2\}](x)X\rangle = N\alpha_1/8\pi m^2\,\langle \mathcal{N}[(n_{\xi\eta}, n)](x)X\rangle +$$

$$+\sqrt{N}(1-\alpha_0)/4\pi m^2\,\langle\tilde{\sigma}(x)X\rangle =$$

$$= [N(1-\alpha_0)/4\pi m^2 - N^2 a\alpha_1/8\pi m^2(1+b)]\langle\sigma(x)X\rangle - \tag{3.2}$$

$$- N\alpha_1/8\pi(1+b)\,\langle \mathcal{N}[\{n^2\}](x)X\rangle + \text{contact terms}$$

The substitution of (3.2) into (3.1) gives the quantum conservation law of the energy-momentum which differs crucially from the classical one [**]:

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[*] In this section the contact terms will not always be written explicitely.

[**] The expressions of this and the following (3.7) conserved currents were suggested by Polyakov on the basis of formal considerations of Lorentz structure and dimension of the introduced ad hoc anomalies, without using the notion of normal products. Note, that in our formalism the "anomalies" arise naturally in the correct quantum equations of motion (2.1).

$$\partial_\eta \langle \mathcal{N}[n^2_\xi](x) X \rangle = \frac{1}{1+b+\frac{N}{8\pi}\alpha_1}\left(\frac{N^2 a\alpha_1}{8\pi(1+b)} + \frac{N}{4\pi}\alpha_0 - 2im^2\tilde{\Gamma}_{\sigma\sigma}(0)\right)\partial_\xi \langle \sigma(x) X \rangle - \tag{3.3}$$

$$- \frac{2i}{1+b}\left(1 - \frac{N\alpha_1}{16\pi(1+b+\frac{N\alpha_1}{8\pi})}\right)\sum_{l=1}^{L}\delta(x-x_l)\langle \partial_\xi n(x) \hat{X}^l \rangle$$

B. We shall search for the first nontrivial conserved current $J^{(3)}_\xi$ (which leads to $\sum\limits_{in} p^3_{l,\xi} - \sum\limits_{ont} p^3_{l,\xi} = 0$) according to the general recipe (0.7) as a linear combination of two operators: $\mathcal{N}[n^2_{\xi\xi}]$ and $\mathcal{N}[(n^2_\xi)^2]$. To this end we consider the following two equations:

$$\partial_\eta \langle \mathcal{N}[n^2_{\xi\xi}](x) X \rangle = - \frac{1}{1+b}\partial_\xi \langle \mathcal{N}[\sigma n^2_\xi - m^2(n^2_\xi - \{n^2_\xi\})](x) X \rangle + \tag{3.4}$$

$$+ \frac{3}{1+b}\langle \mathcal{N}[n^2_\xi \partial_\xi \sigma](x) X \rangle + \text{contact terms}$$

$$\partial_\eta \langle \mathcal{N}[(n^2_\xi)^2](x) X \rangle = \frac{4m^2}{1+b}\langle \mathcal{N}[n^2_\xi(n_\xi, n - \{n\})](x) X \rangle + \tag{3.5}$$

$$+ \text{contact terms}$$

We apply the Zimmermann's identities to the r.h.s. of (3.5) (fig.6). A number of graph cancellations takes place. In fig.6 we have left only those which give nontrivial contributions. The result is:

$$m^2 \langle \mathcal{N}[n^2_\xi(n_\xi, n - \{n\})](x) X \rangle = N f_1 \langle \mathcal{N}[n^2_\xi \partial_\xi \sigma](x) X \rangle + \tag{3.6}$$

$$+ f_2 \partial^3_\xi \langle \sigma(x) X \rangle + f_3 \partial_\xi \langle \mathcal{N}[n^2_\xi \sigma](x) X \rangle + \text{contact terms}$$

The coefficients f_1, f_2, f_3 depend on μ^2/m^2 and are given explicitely in terms of $\langle \cdots \rangle^{prop}$ (see fig.6). For example, the leading order in $1/N$ of f_1 is: $f_1 = -1/8\pi + O(1/N)$.

Thus we see that the total derivatives in ξ and η of the corresponding Green's functions in eqs.(3.4) and (3.5) after accounting for (3.6) are expressed only in terms of $\langle \mathcal{N}[n^2_\xi \partial_\xi \sigma] X \rangle$

and contact terms. Therefore, substituting into (3.4) the result for the composite operator Green's function $\langle \mathcal{N}[n^2_\xi \partial_\xi \sigma] X \rangle$ from (3.5), (3.6) we get the first nontrivial consevation law:

$$\langle \mathcal{N}[\tfrac{1}{2}\partial_\eta J^{(3)}_\xi](x) X \rangle + \langle \mathcal{N}[\tfrac{1}{2}\partial_\xi J^{(3)}_\eta](x) X \rangle + \text{contact terms},$$

$$\mathcal{N}[J^{(3)}_\xi] = \mathcal{N}[n^2_{\xi\xi} - 3/2 N f_1 (n^2_\xi)^2], \tag{3.7}$$

$$\mathcal{N}[J^{(3)}_\eta] = \frac{1}{1+b}\left\{\frac{1}{2}\left(1 + \frac{2f_3}{Nf_1} - f'_1\right)\mathcal{N}[n^2_\xi \sigma] + \left(\frac{f_2}{Nf_1} - f'_2\right)\mathcal{N}[\partial^2_\xi \sigma]\right\}$$

The coefficients f'_1 and f'_2 originate in the Zimmermann's identity:

$$m^2 \langle \mathcal{N}[(n^2_\xi - \{n^2_\xi\})](x) X \rangle = \langle (f'_1 \mathcal{N}[n^2_\xi \sigma] + f'_2 \mathcal{N}[\partial^2_\xi \sigma])(x) X \rangle + \tag{3.8}$$

$$+ \text{contact terms}$$

C. Now we come to the construction of $J^{(5)}$. Consider the derivatives in η of the relevant four composite operator Green's functions:

$$\partial_\eta \langle \mathcal{N}[n^2_{\xi\xi\xi}](x) X \rangle = \frac{m^2}{1+b}\partial_\xi \langle \mathcal{N}[n^2_{\xi\xi} - \{n^2_{\xi\xi}\}](x) X \rangle + \frac{5}{1+b}\langle \mathcal{N}[\partial_\xi \sigma n^2_{\xi\xi} -$$

$$- \partial^3_\xi \sigma n^2_\xi)](x) X \rangle - \frac{1}{1+b}\partial_\xi \langle \mathcal{N}[\sigma n^2_{\xi\xi} - 3\partial^2_\xi \sigma n^2_\xi + 2\partial_\xi(n^2_\xi \partial_\xi \sigma)](x) X \rangle +$$

$$+ \text{contact terms} \tag{3.9}$$

$$\partial_\eta \langle \mathcal{N}[n^2_{\xi\xi} n^2_\xi](x) X \rangle = - \frac{1}{2(1+b)}\partial_\xi \langle \mathcal{N}[\sigma(n^2_\xi)^2](x) X \rangle +$$

$$+ \frac{5}{2(1+b)}\langle \mathcal{N}[\partial_\xi \sigma (n^2_\xi)^2](x) X \rangle + \frac{2m^2}{1+b}\langle \mathcal{N}[n^2_\xi(n^2_{\xi\xi\xi}, n_\xi - \{n_\xi\})](x) X \rangle +$$

$$+ \frac{2m^2}{1+b}\langle \mathcal{N}[n^2_{\xi\xi}(n_\xi, n - \{n\})](x) X \rangle + \text{contact terms} \tag{3.10}$$

$$\partial_\eta \langle \mathcal{N}[(\partial_\xi(n_\xi^2))^2](x)X\rangle = \frac{4m^2}{1+b}\langle \mathcal{N}[\partial_\xi n_\xi^2 \partial_\xi(n_\xi, n-\{n\})](x)X\rangle +$$
$$+\text{contact terms} \tag{3.11}$$

$$\partial_\eta \langle \mathcal{N}[(n_\xi^2)^3](x)X\rangle = \frac{6m^2}{1+b}\langle \mathcal{N}[(n_\xi^2)^2(n_\xi, n-\{n\})](x)X\rangle +$$
$$+\text{contact terms} \tag{3.12}$$

Using again the Zimmermann's identities to express the anisotropic normal products we obtain that r.h.s. of (3.9)-(3.12) have the form;

$$\partial_\xi\langle \mathcal{N}[\dots]X\rangle + g_1\langle \mathcal{N}[\partial_\xi\sigma n_{\xi\xi}^2](x)X\rangle + g_2\langle \mathcal{N}[\partial_\xi\sigma(n_\xi^2)^2](x)X\rangle +$$
$$+g_3\langle \mathcal{N}[\partial_\xi^3\sigma n_\xi^2](x)X\rangle + \text{contact terms}$$

where the coefficients g_1, g_2 and g_3 are given explicitely in terms of the corresponding $\langle\dots\rangle^{prop}$. Therefore, expressing $\langle \mathcal{N}[\partial_\xi\sigma n_{\xi\xi}^2](x)X\rangle$, $\langle \mathcal{N}[\partial_\xi\sigma(n_\xi^2)^2](x)X\rangle$ and $\langle \mathcal{N}[\partial_\xi^3\sigma n_\xi^2](x)X\rangle$ from three of the eqs.(3.9)-(3.12) and substituting the result in the remaining one we get the next nontrivial conservation law (responsible for the conservation of the fifths powers of the initial and final momenta):

$$1/2\,\partial_\eta\langle \mathcal{N}[J_\xi^{(5)}](x)X\rangle + 1/2\,\partial_\xi\langle \mathcal{N}[J_\eta^{(5)}](x)X\rangle =$$
$$=\text{contact terms},$$

$$\mathcal{N}[J_\xi^{(5)}] = \mathcal{N}[n_{\xi\xi\xi}^2] + 1/N\, c_1\mathcal{N}[n_\xi^2 n_{\xi\xi}^2] + 1/N\, c_2\mathcal{N}[(\partial_\xi(n_\xi^2))^2] +$$
$$+1/N\, c_3\mathcal{N}[(n_\xi^2)^3],$$

$$\mathcal{N}[J_\eta^{(5)}] = d_1\mathcal{N}[\partial_\xi^2\sigma n_\xi^2] + d_2\mathcal{N}[\partial_\xi\sigma\partial_\xi(n_\xi^2)] + d_3\mathcal{N}[\sigma\partial_\xi^2 n_\xi^2] +$$
$$+d_4\mathcal{N}[\sigma(n_\xi^2)^2] + d_5\mathcal{N}[\sigma n_{\xi\xi}^2] + m^2\mathcal{N}[d_6\partial_\xi^2 n_\xi^2 + d_7(n_\xi^2)^2 + d_8 n_{\xi\xi}^2]$$

Here, as before, all coefficients are expressed in terms of $\langle\dots\rangle^{prop}$ and their explicit form could be found from the corresponding Zimmermann's identities.

4. Renormalization Group and Infinite Set of Conservation Laws of the Chiral Field

To simplify the analysis of higher conserved currents ($K \geqslant 3$) it is convinient to consider the integrated over x relations (0.8):

$$\sum_{\alpha=1}^{\rho(K)} f_{b\alpha}(\mu^2/m^2)\int d^2x\,\langle[\partial_\xi n;\sigma]_{2K+1}^{\alpha}(x)X\rangle =$$
$$= -\sum_{A=1}^{M(K)} h_{bA}(\mu^2/m^2)\sum_{l=1}^{L}\langle \mathcal{N}[Q_{2K+1}^A](x_l)\hat{X}^l\rangle,\quad b=1,\dots,m(K) \tag{4.1}$$

First we shall describe the general scheme of application of renormalization group equations (2.8) to derive the conservation laws. Then we shall illustrate it in the construction of $J^{(7)}$.

It is easy to check that for $K \geqslant 3$, $m(K) < \rho(K) < 2m(K)$ so (4.1) does not contain sufficient equations for eliminating the integrated "nontotal derivative" terms $\int d^2x\langle[\partial_\xi n;\sigma]_{2K+1}^{\alpha}(x)X\rangle$ in the l.h.s., unlike the case $K=0,1,2$ (see sec.3.). It may happen that the rank $r(K)$ of the rectangular matrix $f_{b\alpha}$ ($\rho(K)\times m(K)$) is: $r(K) < 2m(K) - \rho(K)$. This would imply at least one homogeneous linear relation between the contact terms in (4.1), i.e. an integrated consevation law. The computation of $r(K)$ involves the knowledge of $f_{b\alpha}$ to all orders in $1/N$ (we remind that $f_{b\alpha}$ arise explicitely from the corresponding Zimmermann's identities), which, unfortunately, is technically impossible. Therefore another approach based on the chiral renormalzation group (cf.sec.2) is adopted.

We apply the renormalization group operator:

$$\Delta_{rg} \equiv \mu^2\partial/\partial\mu^2 + \beta(\gamma)\partial/\partial\gamma + L\zeta(\gamma)$$

to both sides of eq.(4.1) and we (2.8) to arrive at a new set of $m(K)$ linear algebraic equations for $\int d^2x\langle[\partial_\xi n;\sigma]_{2K+1}^{\alpha}(x)X\rangle$:

$$\sum_{\alpha=1}^{\rho(K)} f'_{b\alpha}(\gamma)\int d^2x\,\langle[\partial_\xi n;\sigma]_{2K+1}^{\alpha}(x)X\rangle = -\sum_{A=1}^{M(K)} h'_{bA}(\gamma)\sum_{l=1}^{L}\langle \mathcal{N}[Q_{2K+1}^A](x_l)\hat{X}^l\rangle$$

$$f'_{b\alpha}(\gamma) = \sum_{\alpha'=1}^{\rho(K)}\{[\beta(\gamma)\partial/\partial\gamma\, f_{b\alpha'}(\gamma) - \zeta(\gamma)|P_{\alpha'}|f_{b\alpha'}(\gamma)]\delta_{\alpha'\alpha} - f_{b\alpha'}(\gamma)\zeta_{\alpha'\alpha}(\gamma)\}$$

$$h'_{bA}(\gamma)=\sum_{A'=1}^{M(\kappa)}\{[\beta(\gamma)\frac{\partial}{\partial\gamma}h_{bA'}(\gamma)-\zeta(\gamma)(|Q^{A'}|-1)h_{bA'}(\gamma)-\sum f_{b\alpha}(\gamma)z_{\alpha A'}]\delta_{AA'}-h_{bA'}(\gamma)\tilde{\zeta}(\gamma)\} \tag{4.2}$$

$$P_\alpha\equiv\int d^2x\langle[\partial_\xi n,\sigma]^\alpha(x)X\rangle,\quad Q_A\equiv i\sum_\ell\langle\mathcal{N}[Q^A](x_\ell)\hat{X}\rangle$$

where $\zeta_{\alpha\alpha'}$, $\tilde{\zeta}_{AA'}$ and $z_{\alpha A'}$ are respectively the matrices of anomalous dimensions of P_α , Q_A and P_α with respect to Q_A . Note, that the matrices $\zeta_{\alpha\alpha'}$, $\tilde{\zeta}_{AA'}$ and $z_{\alpha A'}$ are finite-dimensional, due to chirality (1.5), inspite of the fact that the canonical dimension of the n -field is zero.

Solving the $2m(\kappa)$ eqs.(4.1), (4.2) in $1/N$ perturbation theory for P_α we get $2m(\kappa)-\rho(\kappa)$ homogeneous linear relations between $M(\kappa)$, $(M(\kappa)>\rho(\kappa))$, integrated contact terms:

$$\sum_{A=1}^{M(\kappa)}h_A^{(j)}\sum_{\ell=1}^{L}\langle\mathcal{N}[Q^A_{2\kappa+1}](x_\ell)\hat{X}^\ell\rangle=0,\ j=1,\dots,2m(\kappa)-\rho(\kappa) \tag{4.3}$$

After applying the LSZ reduction formulae and using $\langle\mathcal{N}[Q^A_{2\kappa+1}]n\rangle^{amp}\big|_{p^2=m^2_{phys}}=p_\xi^{2\kappa+1}\,q_A\left(\frac{\mu^2}{m^2}\right)$, where "amp" means amputation of the external -line, all eqs.(4.3) lead to the same conservation law: $\sum_{\ell=1}^{L}(p_{\ell,\xi})^{2\kappa+1}$. Eqs.(4.3) can be reformulated as (0.5) with coefficients c_b^j for the corresponding conserved current $J_\xi^{(2\kappa+1)(j)}$ (cf.(0.7)) given recursively in $1/N$ perturbation theory by means of the nonintegrated eqs.(4.1), (4.2). Furthermore, (4.3) are equivalent to:

$$[\hat{Q}^{(2\kappa+1)(j)},n_i(x)]=-i\sum_{A=1}^{M(\kappa)}h_A^{(j)}\mathcal{N}[Q^A_{2\kappa+1}](x)$$

$$\hat{Q}^{(2\kappa+1)(j)}=\int dx^1\mathcal{N}[J_0^{(2\kappa+1)(j)}](x) \tag{4.4}$$

One would expect that all $\hat{Q}^{(2\kappa+1)(j)}$ are linearly dependent: $\hat{Q}^{(2\kappa+1)(j)}=\lambda^{(j)}\hat{Q}^{(2\kappa+1)}$ with $\lambda^{(j)}$ a power series in $1/N$, but we have not proved this, except for the leading order in $1/N$ (see below). The charge $\hat{Q}^{(2\kappa+1)}$ is $\xi\dots\xi$ - component of a Lorentz tensor of rank $2\kappa+1$. We remind that there exists another charge which is $\eta\dots\eta$ - component of a Lorentz tensor of rank $2\kappa+1$, coming from the second infinite series of conseved currents, due to the symmetry $\xi\rightleftarrows\eta$ in the eqs. of motion (2.1). This pair of charges can be viewed as the two independent components of a totally symmetric and traceless Lorentz tensor $\hat{Q}^{(2\kappa+1)}_{\mu_1\dots\mu_{2\kappa+1}}$. Each charge generates by (4.4) a quantum renormalized nonlinear in the chiral field representation of an one-parameter group of automorphisms on the field algebra. As in the case of the quantum Sine-Gordon and massive Thirring models, the general structure of the whole infinite dimensional symmetry group remains up to now unclear.

Now we come to the illustration for the existence of $J^{(7}$. There are eight admissible composite operators $[\partial_\xi n]^b_{(8)}$:

$$\mathcal{N}[n^2_{\xi\xi\xi\xi}],\quad\mathcal{N}[n^2_{\xi\xi}\partial^2_\xi(n^2_\xi)],\quad\mathcal{N}[n^2_\xi n^2_{\xi\xi\xi}],\quad\mathcal{N}[(n^2_\xi)^2n^2_{\xi\xi}],$$

$$\mathcal{N}[(n^2_\xi)^4],\quad\mathcal{N}[n^2_\xi(\partial_\xi n^2_\xi)^2],\quad\mathcal{N}[(\partial^2_\xi n^2_\xi)^2],\quad\mathcal{N}[(n^2_{\xi\xi})^2]$$

Here and below all operators are listed in order corresponding to the successive values of the indices $b=1,2,\dots m(3)=8$; $\alpha=1,\dots,\rho(3)=9$; $A=1,2,\dots,M(3)=24$. There are nine operators of type $\int d^2x[\partial_\xi n;\sigma]^\alpha_{(7)}(x)$ in the l.h.s. of eqs.(4.1) and (4.2):

$$\mathcal{N}[\partial^5_\xi\sigma n^2_\xi],\quad\mathcal{N}[\partial^3_\xi\sigma n^2_{\xi\xi}],\quad\mathcal{N}[\partial^3_\xi\sigma(n^2_\xi)^2],\quad\mathcal{N}[\partial_\xi\sigma n^2_{\xi\xi\xi}],$$

$$\mathcal{N}[\partial_\xi\sigma n^2_\xi n^2_{\xi\xi}],\quad\mathcal{N}[\partial_\xi\sigma(\partial_\xi n^2_\xi)^2],\quad\mathcal{N}[\partial_\xi\sigma(n^2_\xi)^3],$$

$$\mathcal{N}[\sigma(\partial_\xi n^2_\xi)n^2_{\xi\xi}],\quad m^2\mathcal{N}[(\partial_\xi n^2_\xi)n^2_{\xi\xi}]$$

The corresponding operators Q^A, $A = 1,2,\ldots, M(3) = 24$ look like:

$$\partial_\xi^7 n_i,\ \mathcal{N}[\partial_\xi^5 n_i n_\xi^2],\ \mathcal{N}[\partial_\xi^4 n_i \partial_\xi n_\xi^2],\ \mathcal{N}[\partial_\xi^3 n_i \partial_\xi^2 n_\xi^2],\ \mathcal{N}[\partial_\xi^3 n_i (n_\xi^2)^2],$$

$$\mathcal{N}[\partial_\xi^3 n_i n_{\xi\xi}^2],\ \mathcal{N}[\partial_\xi^2 n_i \partial_\xi^3 n_\xi^2],\ \mathcal{N}[\partial_\xi^2 n_i \partial_\xi((n_\xi^2)^2)],\ \mathcal{N}[\partial_\xi^2 n_i \partial_\xi n_\xi^2],$$

$$\mathcal{N}[\partial_\xi n_i \partial_\xi^2 n_{\xi\xi}^2],\ \mathcal{N}[\partial_\xi n_i \partial_\xi^2((n_\xi^2)^2)],\ \mathcal{N}[\partial_\xi n_i \partial_\xi^4 n_\xi^2],\ \mathcal{N}[n_i \partial_\xi n_{\xi\xi\xi}^2],$$

$$\mathcal{N}[n_i \partial_\xi n_\xi^2 n_{\xi\xi}^2],\ \mathcal{N}[n_i \partial_\xi n_{\xi\xi}^2 n_\xi^2],\ \mathcal{N}[n_i (\partial_\xi n_\xi^2) \partial_\xi^2 n_\xi^2],$$

$$\mathcal{N}[n_i \partial_\xi ((n_\xi^2)^3)],\ \mathcal{N}[n_i \partial_\xi^3 n_{\xi\xi}^2],\ \mathcal{N}[n_i \partial_\xi^3((n_\xi^2)^2)],\ \mathcal{N}[n_i \partial_\xi^5 n_\xi^2]$$

The system (4.1) with the aid of the abreviated notations P_α, Q_A introduced in (4.2), reads:

$$7P_1 - 14P_2 + 7P_4 = -2Q_1$$

$$m^2\int d^2x \langle \mathcal{N}[n_\xi^2 \partial_\xi^2 (n_{\xi\xi}, n_\xi - \{n_\xi\})](x) X\rangle + m^2 \int d^2x \langle \mathcal{N}[n_{\xi\xi}^2 \partial_\xi^2 \times$$

$$(n_\xi, n - \{n\})](x) X\rangle + 1/2\, P_3 - 3/4\, P_6 = Q_{14} - Q_4 - Q_7$$

$$m^2\int d^2x \langle \mathcal{N}[n_\xi^2 (n_{\xi\xi\xi}^2, n_{\xi\xi} - \{n_{\xi\xi}\})](x) X\rangle + m^2 \int d^2x \langle \mathcal{N}[n_{\xi\xi\xi}^2 \times$$

$$(n_\xi, n - \{n\})](x) X\rangle + 5/2\, P_5 + P_6 + 1/2\, P_8 = Q_2 + 2Q_3 + Q_4 + Q_{10}$$

$$m^2\int d^2x \langle \mathcal{N}[(n_\xi^2)^2 (n_{\xi\xi}, n_\xi - \{n_\xi\})](x) X\rangle + 2m^2 \int d^2x \langle \mathcal{N}[n_{\xi\xi}^2 n_\xi^2 \times$$

$$(n_\xi, n - \{n\})](x) X\rangle + 7/6\, P_7 = 2Q_{11} - Q_5 - Q_8 \tag{4.5}$$

$$m^2\int d^2x \langle \mathcal{N}[(n_\xi^2)^3 (n_\xi, n - \{n\})](x) X\rangle = Q_{13}$$

$$-m^2\int d^2x \langle \mathcal{N}[(n_\xi^2)^2 \partial_\xi^2 (n_\xi, n - \{n\})](x) X\rangle + m^2 \int d^2x \langle \mathcal{N}[(\partial_\xi n_\xi^2)^2 \times$$

$$(n_\xi, n - \{n\})](x) X\rangle = Q_{12} - Q_{15}$$

(4.5)

$$m^2\int d^2x \langle \mathcal{N}[n_\xi^2 \partial_\xi^4 (n_\xi, n - \{n\})](x) X\rangle = Q_{16}$$

$$m^2\int d^2x \langle \mathcal{N}[n_{\xi\xi}^2 (n_{\xi\xi}, n_\xi - \{n_\xi\})](x) X\rangle + P_5 - 1/2\, P_8 = -Q_6 - Q_9$$

P_9 and the remaining Q_A arise after expanding the anisotropic normal products in the l.h.s. of (4.5).

We shall treat the system (4.5) in the $1/N$ perturbation theory.

A. Leading order calculations and recursion to higher orders in $1/N$

We note that leading orders in $1/N$ for the L-point Green's functions of P_α, Q_A ($L \geqslant 8$ without loss of generality [*]) is:

$$P_\alpha, Q_1 = O(N^{1-L/2}),\ \alpha = 1,2,4; \quad P_\beta, Q_A = O(N^{2-L/2}),\ \beta = 2,3,5,$$

$$6,8,9,\quad A = 2,3,4,6,7,9,10,14,17,22,24; \quad P_7, Q_B = O(N^{3-L/2}),$$

$$B = 5,8,11,12,16,18,19,20,23; \quad Q_C = O(N^{4-L/2}),\ C = 13,21$$

Let us denote by $P_\alpha^{(0)}$, $Q_A^{(0)}$, $f_{b\alpha}^{(0)}$ - the leading order contributions, by $P_\alpha^{(1)}$, $Q_A^{(1)}$, $f_{b\alpha}^{(1)}$ - the next to leading order contributions, etc. of P_α, Q_A, $f_{b\alpha}$. The explicite computation of the lowest order Zimmermann's identities in (4.5) is straightforward and gives (3 typical graphical examples are shown in fig.7):

$$m^2\int d^2x \langle \mathcal{N}[n_{\xi\xi}^2 \partial_\xi^2 (n_\xi, n - \{n\})](x) X\rangle = -N/8\pi\, P_2^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[n_\xi^2 \partial_\xi^2 (n_{\xi\xi}, n_\xi - \{n_\xi\})](x) X\rangle = -N/48\pi\, P_1^{(0)} + O(1/N)$$

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[*] The conservation law: for is a consequence from the already established first nontrivial conserved current (sec.3).

$$m^2\int d^2x \langle \mathcal{N}[n^2_{\xi\xi} n^2_{\xi}(n_{\xi}, n-\{n\})](x) X \rangle = -N/8\pi \, P_5^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[n^2_{\xi\xi\xi}(n_{\xi}, n-\{n\})](x) X \rangle = -N/8\pi \, P_4^{(0)} + O(1/N) \tag{4.6}$$

$$m^2\int d^2x \langle \mathcal{N}[n^2_{\xi}(n_{\xi\xi\xi}, n_{\xi\xi}-\{n_{\xi\xi}\})](x) X \rangle = -N/240\pi \, P_1^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[(n^2_{\xi})^2(n_{\xi\xi}, n_{\xi}-\{n_{\xi}\})](x) X \rangle = -N/48\pi \, P_3^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[(n^2_{\xi})^3(n_{\xi}, n-\{n\})](x) X \rangle = -N/8\pi \, P_5^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[(n^2_{\xi})^2\partial^2_{\xi}(n_{\xi}, n-\{n\})](x) X \rangle = -N/8\pi \, P_3^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[(\partial_{\xi} n^2_{\xi})^2(n_{\xi}, n-\{n\})](x) X \rangle = -N/8\pi \, P_6^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[n^2_{\xi}\partial^4_{\xi}(n_{\xi}, n-\{n\})](x) X \rangle = -N/8\pi \, P_1^{(0)} + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[n^2_{\xi\xi}(n_{\xi\xi}, n_{\xi}-\{n_{\xi}\})](x) X \rangle = -N/48\pi \, P_2^{(0)} + O(1/N)$$

Here $O(1/N)$ stands for the next to leading order contributions. Then the integrated eqs. of motion (4.5) in the lowest order acquire the transparent form:

$$7P_1^{(0)} - 14P_2^{(0)} + 7P_4^{(0)} = -2Q_1^{(0)}$$

$$-1/48\pi \, P_1^{(0)} - 1/8\pi \, P_2^{(0)} + 1/2 \, P_3^{(0)} - 3/4 \, P_6^{(0)} = Q_{14}^{(0)} - Q_4^{(0)} - Q_7^{(0)} \tag{4.7}$$

$$-1/240\pi \, P_1^{(0)} - 5/4 \, P_3^{(0)} - 1/8\pi \, P_4^{(0)} + 5/2 \, P_5^{(0)} + P_6^{(0)} + 1/2 \, P_8^{(0)} =$$
$$= Q_2^{(0)} + 2Q_3^{(0)} + Q_4^{(0)} + Q_{10}^{(0)}$$

$$-1/48\pi \, P_3^{(0)} - 1/4\pi \, P_5^{(0)} + 7/6 \, P_7^{(0)} = 2Q_{11}^{(0)} - Q_5^{(0)} - Q_8^{(0)}$$

$$-1/8\pi \, P_7^{(0)} = Q_{13}^{(0)} \; ; \qquad 1/8\pi \, (P_3^{(0)} - P_6^{(0)}) = Q_{12}^{(0)} - Q_{15}^{(0)}$$

$$-1/8\pi \, P_1^{(0)} = Q_{16}^{(0)} \; ; \qquad -1/48\pi \, P_2^{(0)} + P_5^{(0)} - 1/2 \, P_8^{(0)} = -Q_6^{(0)} - Q_9^{(0)}$$

It is seen that $P_9^{(0)}$ does not appear at this step in eqs.(4.5). On the other hand, direct inspection shows that $\det \|\varsigma^{(0)}_{bc}\| = 0$, $b,c=1,\dots,8$ in eq.(4.7). This immediately implies one homogeneous linear relation between the contact terms present in the r.h.s. of (4.5), i.e., the lowest order conservation law for . At the same time one of $P_c^{(0)}$, e.g., $P_8^{(0)}$ and $P_9^{(0)}$ remain undetermined in terms of .

In the r-th order in $1/N$, we obtain the following recursive relations from (4.5):

$$\sum_{c=1}^{8} \varsigma^{(0)}_{bc} P_c^{(r)} = -\sum_A h^{(0)}_{bA} Q_A^{(r)} - \sum_{s=1}^{r-1}\sum_{d=1}^{9} \varsigma^{(s)}_{bd} P_d^{(r-s)} - \sum_{s=1}^{r}\sum_A h^{(s)}_{bA} Q_A^{(r-s)} \tag{4.8}$$

It is clear that in every order in $1/N$ the vanishing $\det\|\varsigma^{(0)}_{bc}\|$ implies a homogeneous linear relation between the r.h.s. terms of (4.8). Unfortunately, the latter includes lower orders $P_8^{(s)}$, $P_9^{(s)}$ which remain at every step in $1/N$ undetermined by $Q_A^{(s')}$, $s' \leq s$.

B. <u>Renormalization group equations and existence of $J_\mu^{(7)}$ to all orders in $1/N$</u>

For the $1/N$ perturbative treatment of (4.2) the following formulae will be of help:

$$\beta(\gamma) = -\gamma^2/4\pi + \gamma^2/2\pi \, (1 - e^{4\pi/\gamma})\zeta(\gamma) \tag{4.9}$$

$$\zeta(\gamma) = -i/2 \; \mu^2 \partial^2/\partial(p^2)^2 \, \Gamma^{(2)}\Big|_{p^2=\mu^2} = O(1/N) \tag{4.10}$$

$$Z_{\varkappa\lambda}(\gamma) = -2e^{4\pi/\gamma}\zeta(\gamma) Z'_{\varkappa\lambda}(\gamma); \quad Z'_{\varkappa\lambda}(\gamma) = O(1/N) \tag{4.11}$$

(4.9), (4.10) are derived from the renormalization group eq. for $\Gamma^{(2)}$ and the normalization conditions (1.3). In (4.11) $Z_{\varkappa\lambda}$, $Z'_{\varkappa\lambda}$ stand for $\zeta_{\alpha\beta}$, ζ_{AB}, $z_{\alpha A}$ and $\zeta'_{\alpha\beta}$, $\tilde{\zeta}'_{AB}$, $z'_{\alpha A}$, respectively. The Z' matrices were introduced in sec.2 and are given in terms of $\langle\ldots\rangle^{prop}$ (fig.4). We rewrite (4.2) by means of (4.9)-(4.11) and (4.1) in the following form (note, that it is valid for all K):

$$[-\gamma^2/4\pi\zeta(\gamma)\,\partial/\partial\gamma f_{b\alpha}+\gamma^2/2\pi(1-e^{4\pi/\gamma})\partial/\partial\gamma f_{b\alpha}-(|P_\alpha|-4)f_{b\alpha}+2e^{4\pi/\gamma}\zeta'_{\alpha\alpha'}f_{b\alpha'}]P_\alpha=$$

$$[\gamma^2/4\pi\zeta(\gamma)\,\partial/\partial\gamma h_{bA}-\gamma^2/2\pi(1-e^{4\pi/\gamma})\partial/\partial\gamma h_{bA}+(|Q^A|-5)h_{bA}- \tag{4.12}$$

$$-2e^{4\pi/\gamma}f_{b\alpha}z'_{\alpha A}-2e^{4\pi/\gamma}h_{bA'}\tilde{\zeta}_{A'A}]Q_A$$

It is straightforward to deduce that the only tems contributing to the leading order in (4.12) are (again $P_9^{(0)}$ does not appear)

$$\sum_{c=1}^{8}[-\gamma^2/4\pi\zeta^{(1)}\,\partial/\partial\gamma f_{bc}^{(1)}-f_{bc}^{(0)}(|P_c|-4)P_c^{(0)}=-\sum_A h_{bA}^{(0)}(|Q^A|-5)Q_A^{(0)} \tag{4.13}$$

The one-loop graphs from the boxes in fig.7 with one $b^{(1)}$-counterterm insertion contribute to the nonzero $\partial/\partial\gamma f_{bc}^{(1)}$:

$$\partial/\partial\gamma f_{22}^{(1)}=\partial/\partial\gamma f_{34}^{(1)}=1/2\,\partial/\partial\gamma f_{45}^{(1)}=\partial/\partial\gamma f_{57}^{(1)}=-\partial/\partial\gamma f_{63}^{(1)}=\partial/\partial\gamma f_{66}^{(1)}=$$

$$=\partial/\partial\gamma f_{77}^{(1)}=1/8\pi\,\partial/\partial\gamma b^{(1)}=-1/\gamma^2\zeta^{(1)}(\gamma),$$

$$\partial/\partial\gamma f_{21}^{(1)}=\partial/\partial\gamma f_{43}^{(1)}=\partial/\partial\gamma f_{32}^{(1)}=1/48\pi\,\partial/\partial\gamma b^{(1)}=-1/6\gamma^2\zeta^{(1)}(\gamma) \tag{4.14}$$

$$\partial/\partial\gamma f_{31}^{(1)}=1/240\pi\,\partial/\partial\gamma b^{(1)}=-1/30\gamma^2\zeta^{(1)}(\gamma)$$

The relation $\partial/\partial\gamma b^{(1)}=-8\pi\zeta^{(1)}/\gamma^2$ *) used in (4.14) is directly obtained by (4.10) and the normalization conditions (1.3).

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*) We emphasize the crucial role of this relation to obtain the simple and explicite coefficients $f'^{(0)}_{b\alpha}$ in (4.15).

Substituting (4.14) into (4.13) we arrive at the following lowest order eqs.:

$$0=Q_1^{(0)}=Q_4^{(0)}+Q_7^{(0)}-Q_{14}^{(0)}=Q_2^{(0)}+2Q_3^{(0)}+Q_4^{(0)}+Q_{10}^{(0)}=Q_{16}^{(0)}=$$

$$=Q_6^{(0)}+Q_9^{(0)}$$

$$1/48\pi\,P_3^{(0)}+1/4\pi\,P_5^{(0)}-7/6\,P_7^{(0)}=0 \tag{4.15}$$

$$-1/4\pi\,P_7^{(0)}=Q_{13}^{(0)}\;;\qquad 1/4\pi\,(P_3^{(0)}-P_6^{(0)})=0$$

(4.15), together with (4.7), implies: $\sum_A h_{bA}^{(0)}Q_A^{(0)}=0$. However, there is only one of them: $Q_1^{(0)}=0$ which gives, after amputation and going to the mass shell nontrivial conservation law: $\sum_\ell (P_{\ell,\xi})^7=0$ *).

The r-th order in $1/N$ of (4.12) looks analogously to (4.8):

$$\sum_{c=1}^{8}f'^{(0)}_{bc}P_c^{(r)}=-\sum_A h'^{(0)}_{bA}Q_A^{(r)}-\sum_{s=1}^{r}\sum_{\alpha=1}^{9}f'^{(s)}_{b\alpha}P_\alpha^{(r-s)}-\sum_{s=1}^{r}\sum_A h'^{(s)}_{bA}Q_A^{(r-s)} \tag{4.16}$$

It is obvious from the explicitely known $f_{b\alpha}^{(0)}$, $h_{bA}^{(0)}$, $f'^{(0)}_{b\alpha}$, $h'^{(0)}_{b\alpha}$, given by (4.7), respectively (4.15) that eqs.(4.16) are not corollaries (linear combinations) of eqs.(4.8) in each r-th order in $1/N$.. Hence, we obtain in general seven homogeneous linear relations between $Q_A^{(s)}$, $s\leqslant r$, which as already mentioned are suspected to be linearly dependent. Nevertheless, all of them on the mass shell give rise to the same conservation law: $\sum_{\ell=1}^{L}(P_{\ell,\xi})^7=0$ (see comment after (4.3)).

The detailed proof of the existence of the conserved current $J_\mu^{(7)}$ is carried over to the general case, $K>3$, in a straightforward manner. This is possible because of our ability to compute explicitely the leading order graphs, arising from the corresponding Zimmermann's identities and because of the fact that only these graphs are needed to reveal the linear independence of the renormalization group eqs. (4.2) with respect to the in-

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*) We remark that in the leading order in $1/N$ the physical mass is equal to the mass parameter m.

tegrated eqs. of motion (4.1) in every order of the $1/N$ expansion. Some examples, analogous to (4.6) are (B - an arbitrary field monomial) (cf.fig.7):

$$m^2\int d^2x \langle \mathcal{N}[B(n_\xi, n-\{n\})](x)X\rangle = -N/8\pi \int d^2x \langle \mathcal{N}[B\partial_\xi\sigma](x)X\rangle + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[B(n_{\xi\xi}, n_\xi-\{n_\xi\})](x)X\rangle = -N/48\pi \int d^2x \langle \mathcal{N}[B\partial^3_\xi\sigma](x)X\rangle + O(1/N)$$

$$m^2\int d^2x \langle \mathcal{N}[B(n_{\xi\xi\xi}, n_{\xi\xi}-\{n_{\xi\xi}\})](x)X\rangle = -N/240\pi \int d^2x \langle \mathcal{N}[\partial^5_\xi\sigma](x)X\rangle + O(1/N)$$

We would like to emphasize that two principal facts are crucial for the success of the above construction:

1. Peculiarity of Lorentz covariance in two dimensional space-time (light-cone compnents).

2. Chirality (1.5) which, apart from ensuring the strict renormalizability of the theory (nontrivial renormalization group, in particular), renders finite the set of all composite operators of equal Lorentz structure and canonical dimension in the Zimmermann's identities and the renormalization group equations. The latter is not true for $P(\varphi)_2$ scalar superrenormalizable models, e.g., $O(N)$ covariant $(\varphi^4)_2$ theory. One could be easily convinced that this scheme for construction of an infinite set of polynomial quantum conserved currents has to work in two-dimensional strictly renormalizable fermion theories. For example, the only nontrivial strictly renormalizable (in the sense of usual perturbation theory) $U(1)$ invariant fermion model is the massive Thirring model which has already been shown to possess an infinite set of polynomial conservation laws [14-17] by another argument based on the complete integrability of the classical counterpart.

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FIGURE CAPTIONS

Fig.1. Graphical elements of the $1/N$ perturbation theory.

Fig.2. Cutkosky rules for n - and σ-lines.

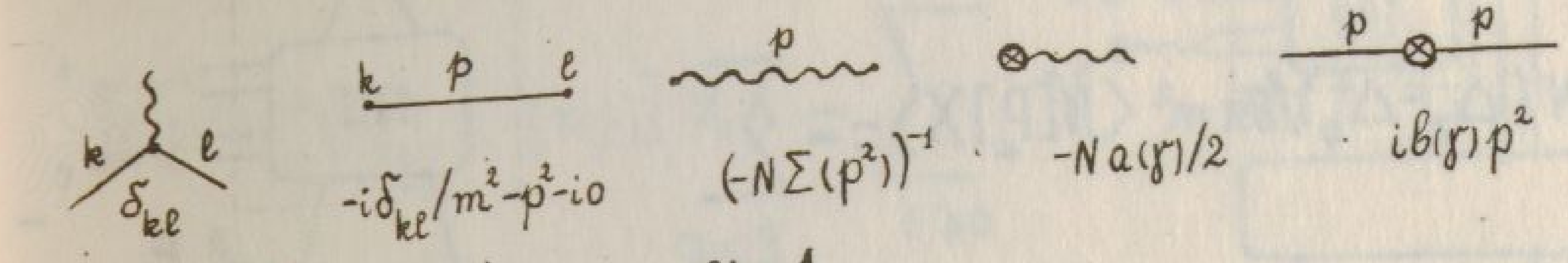
Fig.3. Quantum chiral equations of motion. The bar on a line denotes derivative with respect to x. P is an arbitrary composite operator. The vertical dotted line denotes omission of this line.

Fig.4. Zimmermann's identity for $\Delta_2-\Delta_0$. The difference of two Taylor operators of orders $\omega+2$, ω respectively: $D^{[\omega+2]}+D^{[\omega+1]}\equiv t^{\omega+2}-t^{\omega}$ acts on the 1PI subgraphs in the boxes, where ω is their superficial divergence index. The contribution of $D^{[\omega+1]}$ vanishes due to Lorentz covariance.

Fig.5. Zimmermann's identity for $\langle \mathcal{N}[n^2-\{n^2\}](x)X\rangle$

Fig.6. Zimmermann's identity for $\langle \mathcal{N}[n_\xi^2(n_\xi, n-\{n\})](x)X\rangle$. Summation over isotopic indices i,j is assumed.

Fig.7. Typical lowest order graphs contributing to the Zimmermann's identities (4.6).



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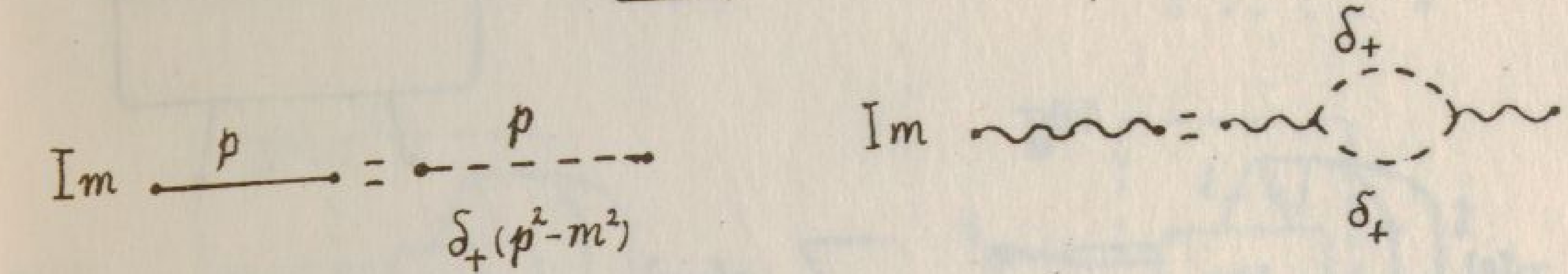

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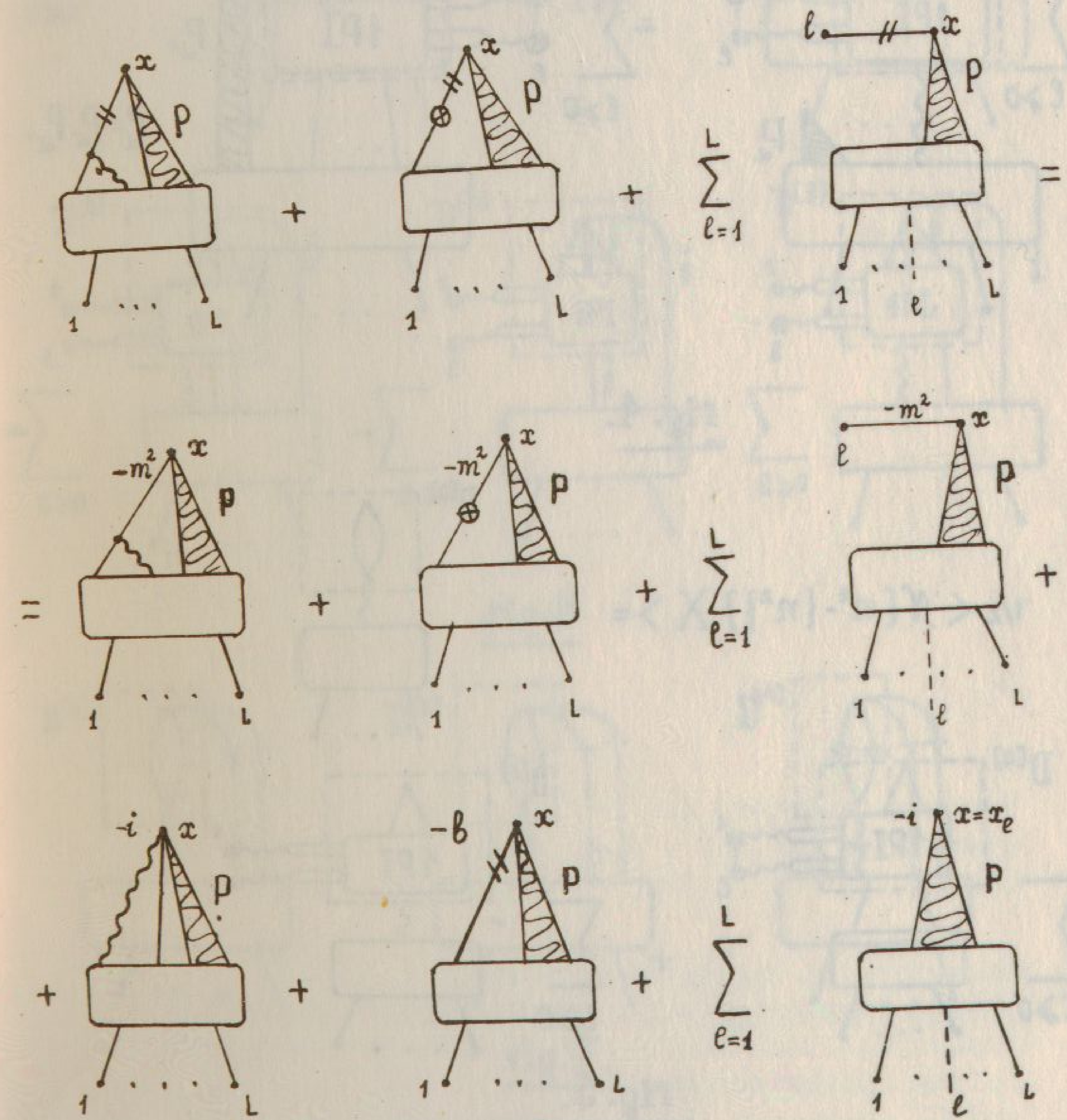

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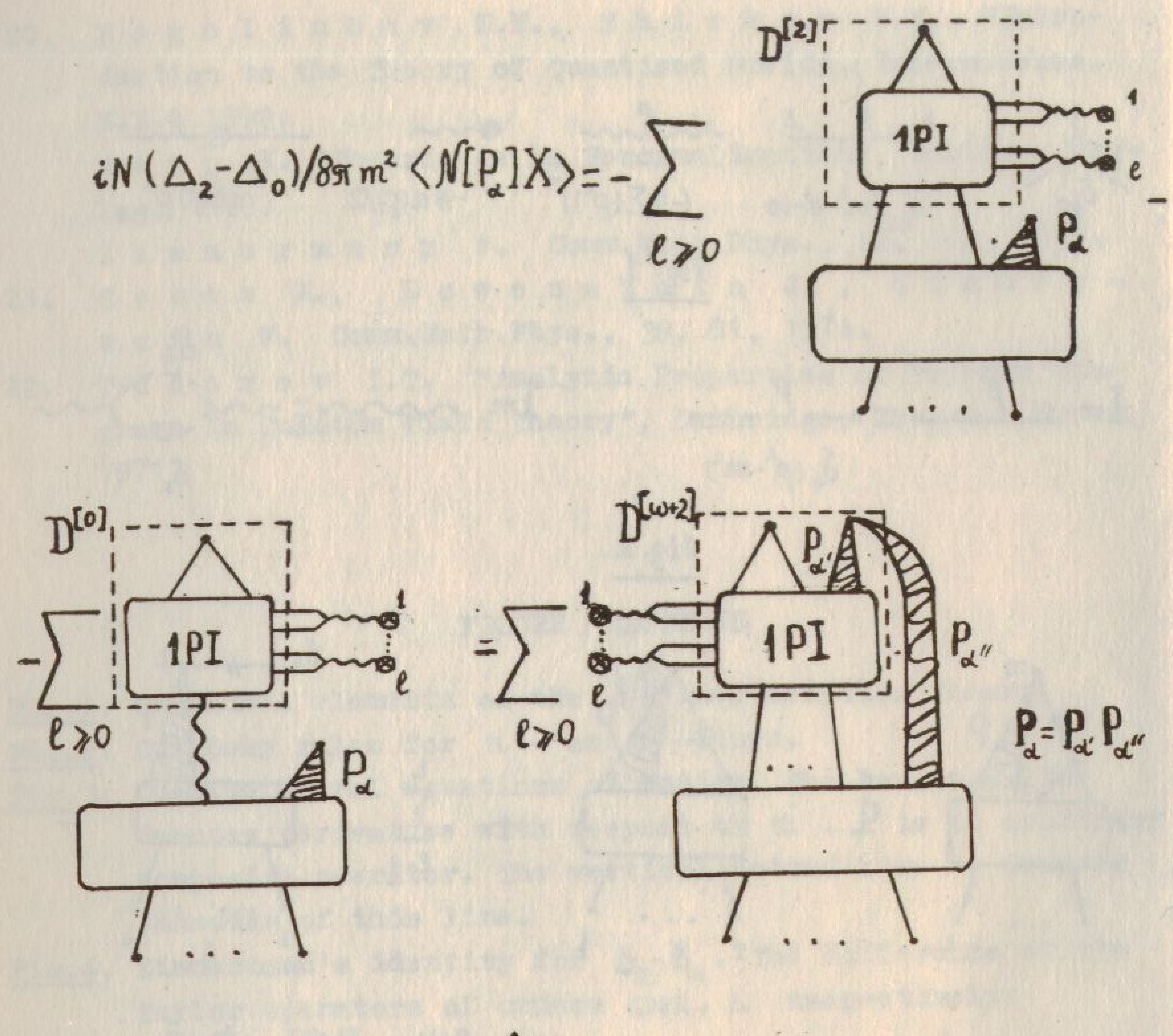

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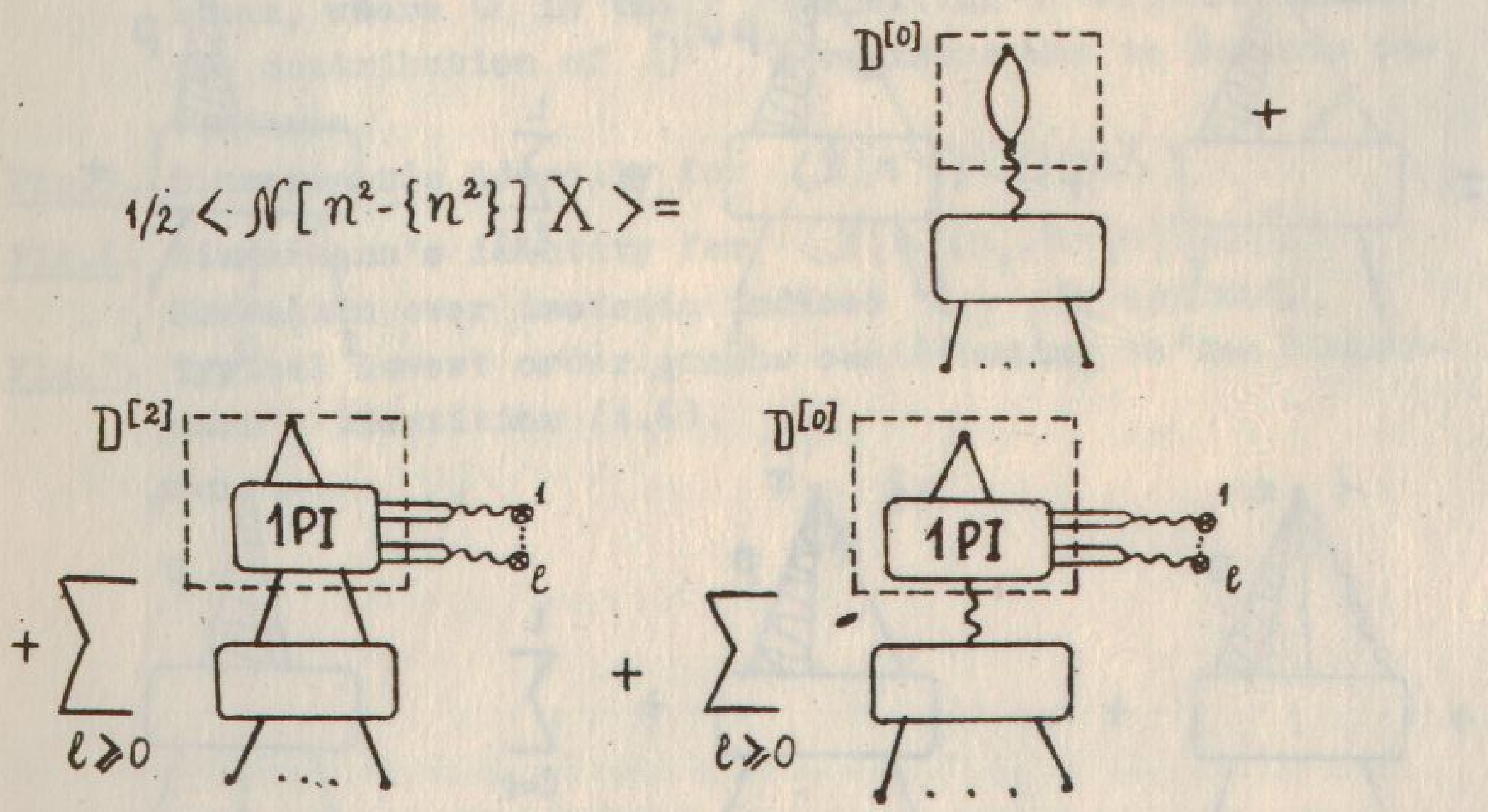

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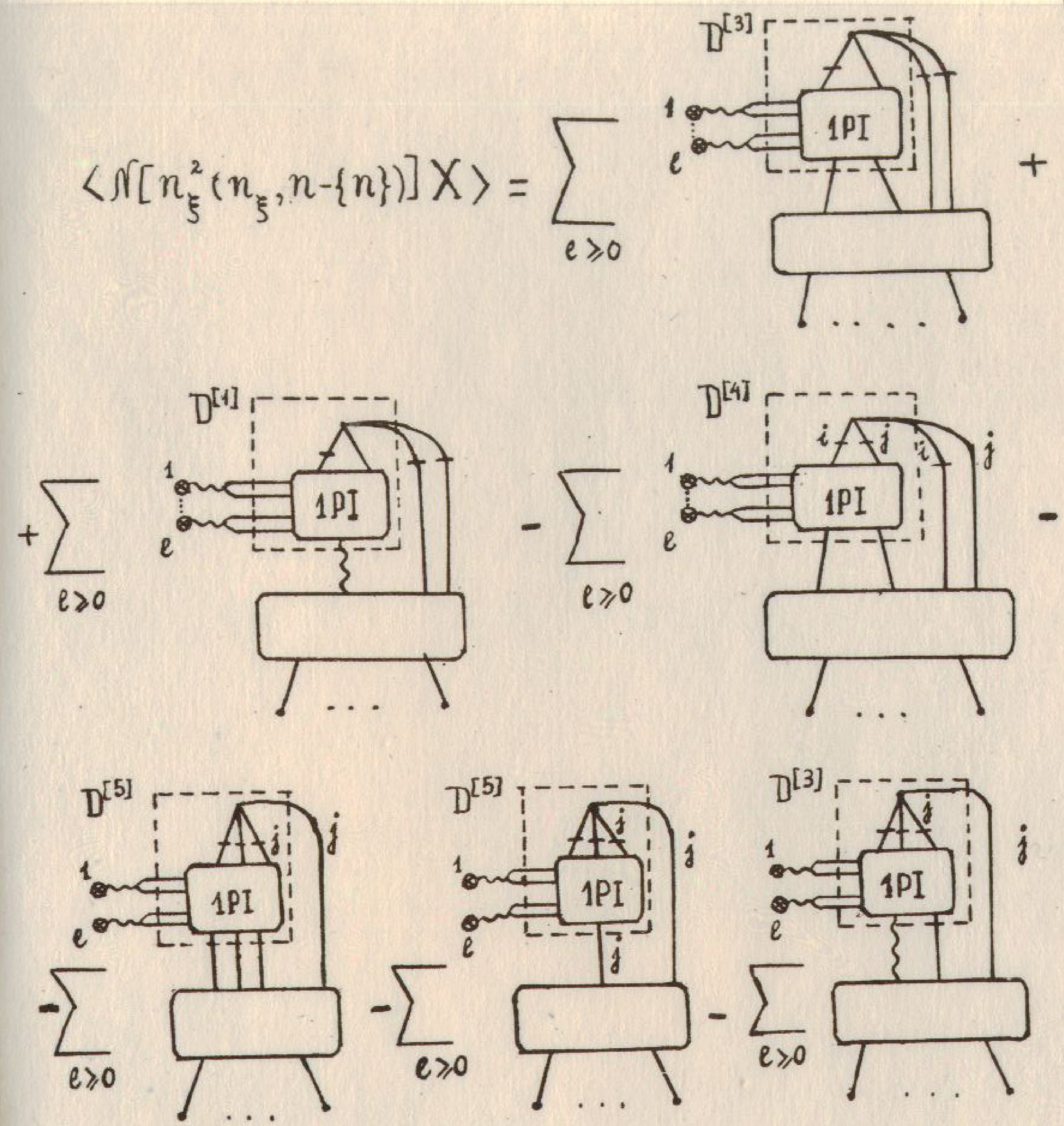

fig.6

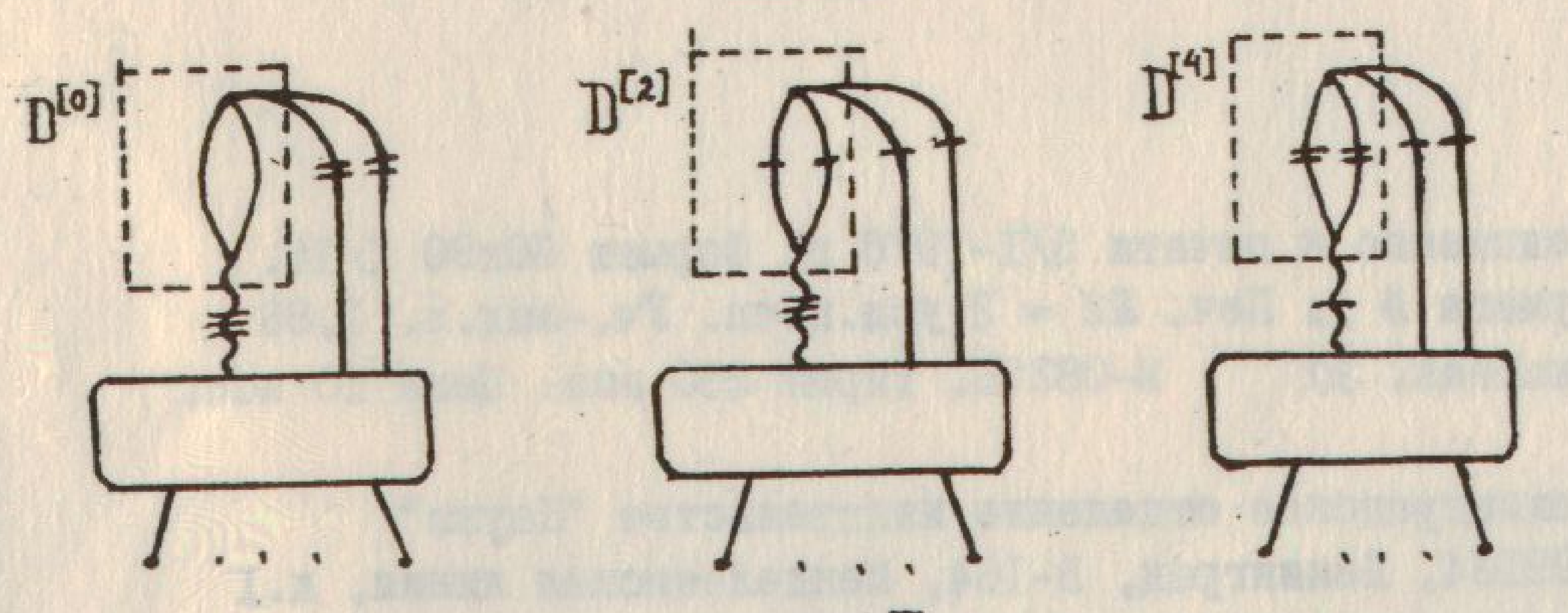

fig.7.

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